Advanced Physical Research
Vol.5, No.3, 2023, pp.123-132

DRIVEN SINE-GORDON EQUATION ON METRIC GRAPHS: A MODEL FOR BRANCHED JOSEPHSON JUNCTION

M.E. Akramov[1], I.N. Askerzade[2,4*], M. Salati[2], D.U. Matrasulov[3]

[1]National University of Uzbekistan, Tashkent, Uzbekistan
[2]Department of Computer Engineering, Ankara University, Ankara, Turkey
[3]Turin Polytechnic University in Tashkent, Tashkent, Uzbekistan
[4]Institute of Physics, Baku, Azerbaijan

Abstract. We consider driven sine-Gordon equation on a metric star graph (Y-junction) by choosing driving potential in the form of periodic monochromatic function. Solutions of the problems are obtained numerically. Transmission of sine-Gordon solitons through the branching point of the graph is analyzed using the contour plots of the solution. Obtained solutions are used for modeling of soliton dynamics electronic properties of branched Josephson junction. Current profile and current-voltage characteristics are plotted and analyzed. The approach used for the case of the star-branched Josephson junction can be extended to arbitrary branching architectures.

Keywords: *Josephson junction, sine-Gordon solitons, metric graphs.*

Corresponding Author: *I.N. Askerzade, Department of Computer Engineering, Ankara University, 06100, Ankara, Turkey, Tel.:903122033300/1750, e-mail: imasker@eng.ankara.edu.tr*

Received: *27 May 2023;* ***Accepted:*** *12 July 2023;* ***Published:*** *03 October 2023.*

1. Introduction

Fabrication of the functional materials having high resource saving properties and maximum of device optimization and miniaturization, is a key priority for modern condensed matter physics and material science. Solving such a task requires deep understanding and manipulation by underlying fundamental phenomena. An important task in this context is reducing the dimension, or using low-dimensional functional materials for device fabrication. Remarkable feature of such dimensional reduction is possibility for tuning of material and device properties. Especially, such a reduction is effective in case of quasi-one-dimensional branched structures and networks. Playing with the device branching (architecture) topology is a powerful tool for tuning of the electronic, optical, thermal and mechanical properties in low-dimensional functional materials having branched structure. Transmission quasiparticles and waves through the branching points (nodes) of the structure and providing minimum of backscattering in such transmission allows to reduce energy and signal loss in utilization of branched low-dimensional materials in different devices. Among other devices fabricated on the basis of low-dimensional functional materials, Josephson junctions attracted much attention during past few decades. Potential use of Josephson junctions as the basic functional unit of SQUID, practical applications in creating of qubits and other possible applications in

How to cite (APA):
Akramov, M.E., Askerzade, I.N., Salati, M., & Matrasulov, D.U. (2023). Driven sine-Gordon equation on metric graphs: A model for branched Josephson junction. *Advanced Physical Research, 5*(3), 123-132.

nanoelectronics in quantum technology, made them one of the central elements of the emerging technologies. Different versions of Josephson junction based devices have been considered during the past few decades. Modeling solitons in terms of sine-Gordon equation is discussed in the (Barone & Paternò, 1982; Askerzade *et al.*, 2017; Maraver *et al.*, 2014; Likharev, 1986). One of the interesting versions of such device is so called branched Josephson junction or Josephson junction network (Susanto *et al.*, 2004; Matrasulov *et al.*, 2020; Burioni *et al.*, 2006; Burioni *et al.*, 2005). Such devices provide powerful tool for tuning of different quantum and topological phenomena underlying Josephson junction physics (Askerzade & Salati, 2022; Askerzade, 2015; Askerzade & Kornev, 1994). In particular, by choosing proper device branching architecture one can achieve needed regime of sine-Gordon soliton dynamics, Josephson current and current-voltage characteristics. In this paper we address the problem of periodically driven branched Josephson junction which can appear, e.g., in multiterminal Josephson junctions (Amin *et al.*, 2002a; Amin *et al*, 2002b; Amin *et al.*, 2001; Heck *et al.*, 2014; Golikova *et al.*, 2014; Riwar *et al.*, 2016; Zazunov *et al.*, 2017; Nowak *et al.*, 2019; Pankratova *et al.*, 2020; Melin, 2022; Amet *et al.*, 2022; Gavensky *et al.*, 2023). Dynamics of solitons in such structures can be modelled in terms of a sine-Gordon equation on metric graphs, where the kink-solitons appear as the phase difference in Josephson junction branches. It should be noted that modeling solitons using the metric graph based approach attracted much interest in the literature during the past two decades (Susanto *et al*., 2004; Sobirov *et al.*, 2016; Sabirov *et al.*, 2018; Babajanov *et al.*, 2018; Matrasulov *et al.*, 2020; Caputo & Dutykh, 2014). Discussion of the sine-Gordon equation on metric graphs describing the phase difference in a 0-π and the specific case of a tricrystal boundary with a π Josephson junction as one of the three arms can be found in Ref. (Susanto *et al*., 2004). Integrability and the soliton solutions of the sine-Gordon equation on metric graphs was considered in (Sobirov *et al*., 2016), where existence of infinitely many conservation laws was proven and exact soliton solution were obtained. It was shown that when the problem is integrable, transmission of the sine-Gordon solitons through the network vertices is reflectionless. Stationary sine-Gordon equation on finite metric graphs have been studied in the Refs. (Sabirov et al. 2018; Babajanov et al. 2018; Matrasulov et al. 2020; Sabirov, Sobirov, Babajanov & Matrasulov, 2013). Static sine-Gordon solitons in branched Josephson junctions were considered in the Ref. (Matrasulov *et al.*, 2020). We note that the driven sine-Gordon equation on a line is studied in the Refs. (Gul *et al.*, 2018; Pankratov, 2002; Jagtap *et al.*, 2017; Bennett *et al.*, 1982; Malomed, 1989). Here we use a time-dependent sine-Gordon equation on graphs with external, time-periodic potential for modeling of driven Josephson junction. By solving the problem numerically the time-dependent driven sine-Gordon equation, we compute the current-voltage characteristics of the device.

The paper is organized as follows. In the next section we introduce the driven sine-Gordon equation on the star graph. In Section 3, we present the numerical results, current-voltage characteristics and discussion of the results. Section 4 presents some concluding remarks.

2. Driven sine-Gordon equation on a star graph

Driven, or modified sine-Gordon equation on a real line can be written as Gul et al. (2018)

$$\partial_t^2\psi(x,t) - \partial_x^2\psi(x,t) + \sin[\psi(x,t)] = \gamma + \lambda\partial_t\psi(x,t) + f\cos(\omega t), \quad (1)$$

where $\gamma,\ f,\ \omega$ are constants.

Eq. (1) described phase difference in a Josephson junction driven by external time-periodic (e.g., AC) field. Unlike to standard (integrable) sine-Gordon equation, it does not approve analytical soliton solutions. Therefore, one needs to solve it numerically. Here we consider driven sine-Gordon equation on a metric graph, by focusing in star, i.e. Y −junction branched graph. Namely, we consider the star graph with three semi-infinite bonds b_j (see, Fig. 1), for which a coordinate x_j is assigned. Choosing the origin of coordinates at the vertex, 0 for bond b_1 we put $x_1 \in (-\infty, 0]$ and for $b_{2,3}$ we fix $x_{2,3} \in [0, +\infty)$.

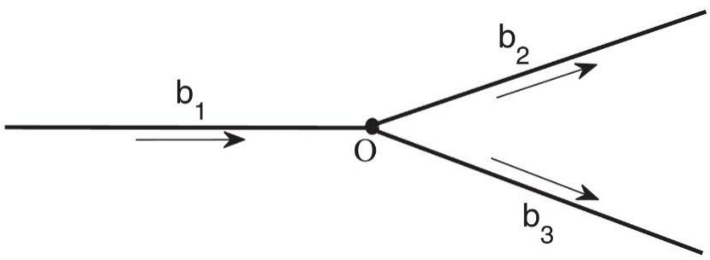

Fig. 1. Basic star graph

Then driven sine-Gordon equation can be written on each bond of the star graph as

$$\begin{aligned}\partial_t^2\psi_j(x,t) - \alpha_j^2\partial_x^2\psi_j(x,t) + \beta_j^2 \sin\left[\psi_j(x,t)\right] \\ = \gamma_j + \lambda_j\partial_t\psi_j(x,t) + f_j\cos(\omega_j t),\end{aligned} \quad (2)$$

Vertex boundary conditions which can be derived from fundamental conservation laws, such as charge and energy conservation, are given as (Sobirov *et al.*, 2016):

$$\begin{aligned}\psi_1(x,t)|_{x=0} = \psi_2(x,t)|_{x=0} = \psi_3(x,t)|_{x=0}, \\ a_1\partial_x\psi_1(x,t)|_{x=0} = a_2\partial_x\psi_2(x,t)|_{x=0} + a_3\partial_x\psi_3(x,t)|_{x=0}.\end{aligned} \quad (3)$$

From vertex boundary conditions (3) one can derive the following constraint:

$$a_j = \frac{\alpha_j}{\beta_j}, \qquad \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3}, \quad (4)$$

which provides condition to be fulfilled by the solution of Eq. (1) to fulfill the vertex boundary conditions given by Eq. (2), i.e. condition for fulfilling by the solution of driven sine-Gordon equation on a line to that star on a graph. Assuming that the constraint (4) is fulfilled, we solved the problem given by Eqs. (2) and (3). For numerical solution we used fourth-order Runge-Kutta method for the system of second order differential equations in Eq. (2). In order to apply this method one needs to rewrite Eq. (2) as in the form

$$\begin{aligned} \partial_t \psi_j(x,t) &= \phi_j(x,t), \\ \partial_t \phi_j(x,t) - \alpha_j^2 \partial_x^2 \psi_j(x,t) + \beta_j^2 \sin\left[\psi_j(x,t)\right] &\\ = \gamma_j + \lambda_j \partial_t \psi_j(x,t) + f_j \cos(\omega_j t),& \end{aligned} \quad (5)$$

The initial condition is imposed in the form of anti-kink solution of sine-Gordon equation on a graph considered in Sobirov et al. (2016) which is given by

$$\psi_j(x,0) = 4\arctan\left[\exp\left(-\frac{\frac{\beta_j}{\alpha_j}(x-x_0)}{\sqrt{1-v^2}}\right)\right],$$

where v is the velocity of the anti-kink. For all calculations, we choose the parameters α_j and β_j as fulfilling the constraint in Eq. (4), namely $\alpha_1 = 1$, $\alpha_2 = 0.6$, $\alpha_3 = 0.4$ and $\beta_1 = \beta_2 = \beta_3 = 1$.

Figs. (2) and (3) present the plots of numerically obtained solution of the problem given by Eqs. (2) and (3). Dynamics of sine-Gordon (kink) soliton near the branching point can be analyzed from these plots. Namely, transmission of the soliton through the vertex is accompanied by reflection (backscattering). Such a behavior is opposite to what was observed in case of unperturbed sine-Gordon equation on metric graphs, where transmission of kink solitons through the vertex for the case when the constraint in Eq. (4) was reflectionless (Sobirov *et al.*, (2016).

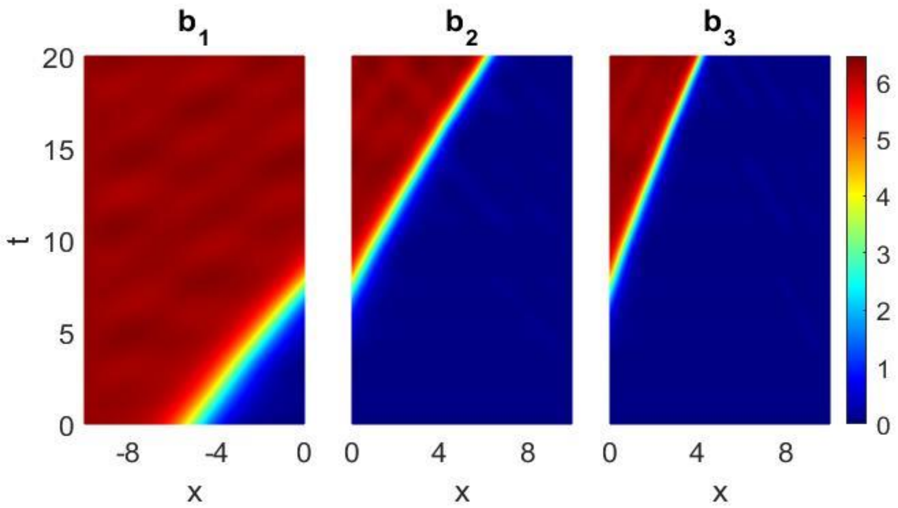

Fig. 2. Contour plot of numerical solution of Eq. (2) for parameters: $\boldsymbol{\gamma_j = 0,\ \lambda_j = 0.05,\ f_j = 0.1,\ \omega_j = 2}$.

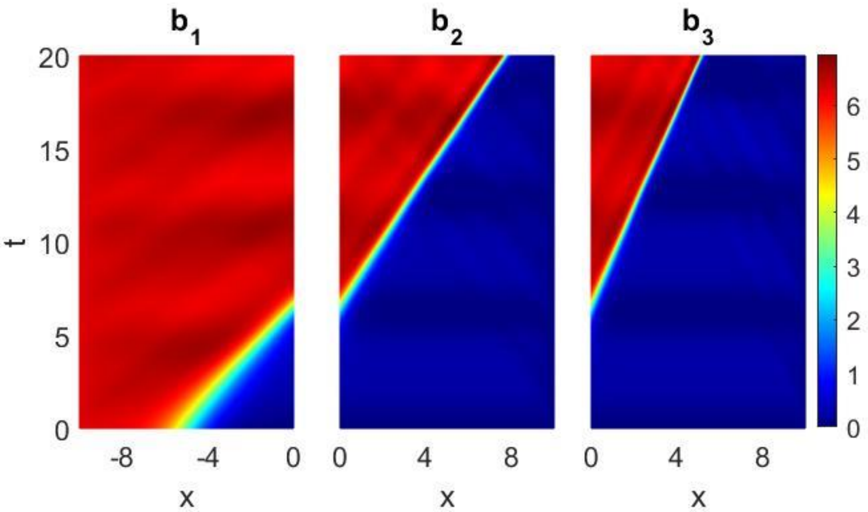

Fig. 3. Contour plot of numerical solution of Eq. (2) for parameters:
$\gamma_j = 0.15,\ \lambda_j = 0.05,\ f_j = 0.1,\ \omega_j = 2.$

Such a difference can be observed by the role of the external driving potential, that makes the problem non-integrable.

3. Branched Josephson junction

Here we apply the results of the previous section to branched Josephson junction. The latter is a device presented in Fig. 4, that consists of planar superconductor sheets, connected each other via the branched normal metal (or insulator) having Y −junction shape with long branches. Such structure can be considered as a version of the so-called multi-terminal Josephson junction considered in the Refs. (Amin *et al.*, 2002a; Amin *et al.*, 2002b; Amin *et al.*, 2001; Riwar *et al.*, 2016; Pankratova *et al.*, 2020). The most closest to our model Josephson junction device was quite recently considered in (Gavensky *et al.*, 2023). The phase difference on each branch such device is described in terms of Eq. (2) and fulfills the vertex boundary conditions given by Eq. (3).

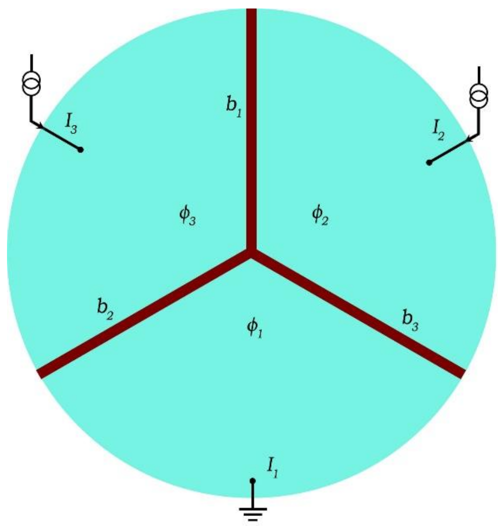

Fig. 4. Sketch for the branched Josephson junction

Therefore the numerical results obtained in the previous section can be used for computing its electronic properties, e.g. the profile of current and the current-voltage characteristics. To do this, we can use relation of the between current and phase difference, as well as relation between voltage and phase difference, which are given by (respectively) (Barone *et al.*, 1971).

$$J_j = J_{0j} \sin\left(\psi_j(x,t)\right),$$
$$V_j = \frac{\hbar}{2e}\partial_t \psi_j(x,t)$$

where J_{0j} is the amplitude of the Josephson current. Fig. 5 presents the plots of profile of the current on each branch of the Josephson junction at different time moments. Crucial change of the current's profile in transmission from the first branch to second and third ones can be clearly seen from the plots. Symmetry between the second and third bonds is caused by the initial condition, which imposed on first bond as incoming sine-Gordon kink at $t = 0$.

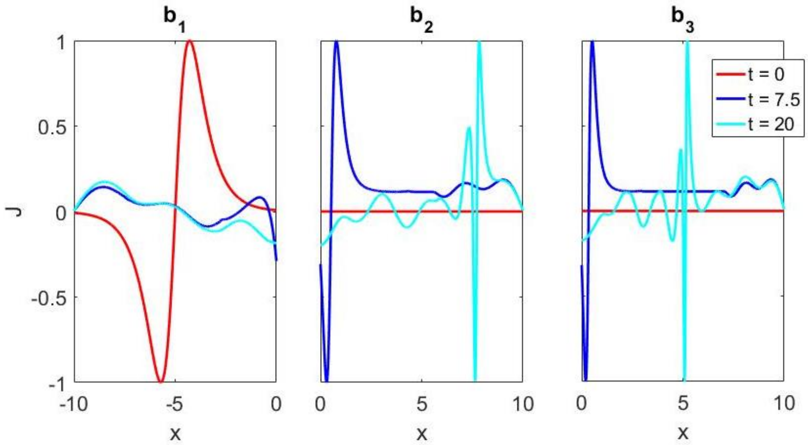

Fig. 5. Current profile on each branch of the Josephson junction presented in Fig. (4) at different time moments. The parameters are chosen as $\boldsymbol{\gamma_j = 0.15,\ \lambda_j = 0.05,\ f_j = 0.1,\ \omega_j = 2}$

In Figs. (6) and (7) demonstrated the current-voltage characteristics on each branch of the star shaped branched Josephson junction (see, Fig. 4).

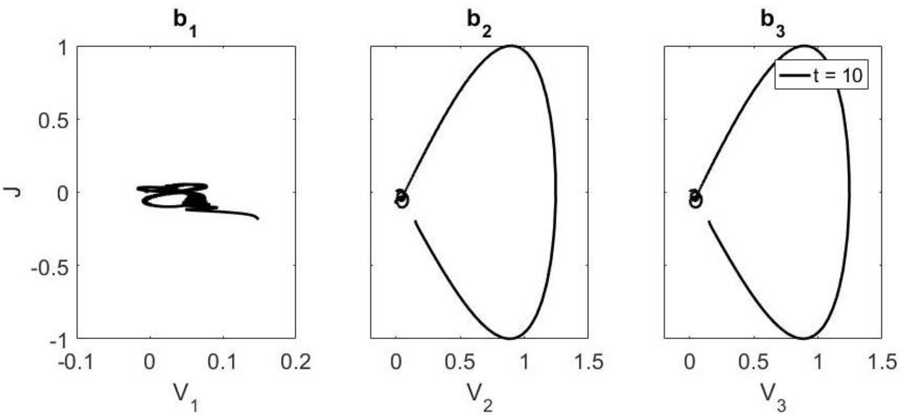

Fig. 6. Current-voltage characteristics on each branch of the Josephson junction presented in Fig. (4) for the parameters: $\boldsymbol{\gamma_j = 0,\ \lambda_j = 0.05,\ f_j = 0.1,\ \omega_j = 2}$.

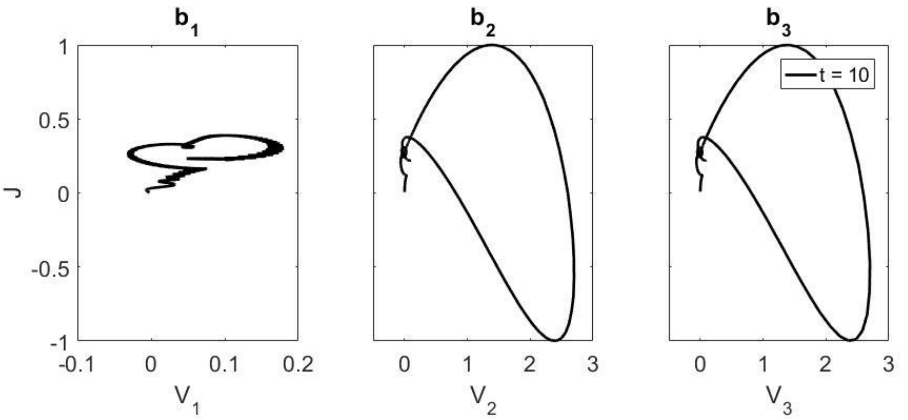

Fig. 7. Current-voltage characteristics on each branch of the Josephson junction presented in Fig. (4) for the parameters: $\gamma_j = 0.15,\ \lambda_j = 0.05,\ f_j = 0.1,\ \omega_j = 2$.

Abrupt change of the shape of the $I - V$ curve can be observed for these plots, too. Such a change can be explained by existence of certain backscattering and dominating (with respect to backscattering) at the junction's branching point. It is clear that by manipulating branch lengths and the sizes of superconducting domains, one can achieve tuning of the current-voltage characteristics.

Especially, such a tuning becomes effective, when one considers more complicated (than the simple $Y -$ junction) branching architectures, such as tree, loop, octagon, etc.

4. Conclusions

In this paper we studied soliton dynamics in branched Josephson junction driven by external time-periodic field. The whole is modelled in terms of modified sine-Gordon equation containing time-periodic potential. By solving sine-Gordon equation numerically we explored soliton dynamics on each branch and transmission of sine-Gordon solitons through the junction's branching point. Current-voltage characteristics of the device is plotted using the obtained solution of the sine-Gordon equation. The model we proposed can be fabricated and experimentally studied by splitting bulk planar superconductor via branched ($Y -$junction) normal metal or insulator. Although the above treatment dealt with star-shaped branched Josephson junction, the approach we used can be applied for arbitrary branching architecture. Such a study is a subject for our forthcoming research that should appear in nearest future. The above results can be used for engineering different micro- and nano-scale devices on the basis of branched Josephson junctions

Acknowledgments

This work is supported by joint grant the Ministry of Innovative Development of Uzbekistan (Ref. Nr. MRT-2130213155) and TUBITAK (Ref. Nr. 221N123).

References

Adami, R., Cacciapuoti, C., Finco, D. & Noja, D. (2011). Fast solitons on star graphs. *Reviews in Mathematical Physics, 23*(4).

Adami, R., Cacciapuoti, C. & Noja, D. (2016). Stable standing waves for a NLS on star graphs as local minimizers of the constrained energy. *Journal of Differential Equations*, 260, 7397.

Adami, R., Serra, E. & Tilli, P. (2017). Negative energy ground states for the L 2-critical NLSE on metric graphs. *Communications in Mathematical Physics*, 352, 387.

Akramov, M., Khashimova, F. & Matrasulov, D. (2023). Discrete nonlocal nonlinear Schrödinger equation on graphs: dynamics of PT-symmetric solitons in discrete networks. *Physics Letters A*, 457, 128555.

Akramov, M., Sabirov, K., Matrasulov, D., Susanto, H., Usanov, S. & Karpova, O. (2022). Nonlocal nonlinear Schrödinger equation on metric graphs: A model for generation and transport of parity-time-symmetric nonlocal solitons in networks. *Physical Review E*, 105, 054205.

Amet, F., Idris, S., McConnell, A., Opatosky, B. & Arnault, E. (2022). Phase dynamics in an AC-driven multiterminal Josephson junction analog. *Physical Review B*, 106, 174509.

Ameya, D. Jagtap, Saha, E., Jithin, D. George, A.S. & Murthy, V. (2017). Revisiting the inhomogeneously driven sine – Gordon equation, *Wave Motion*, 73, 76-85.

Amin, M.H.S., Omelyanchouk, A.N., Blais, A., Maassen, A. van den Brink, Rose, G., Duty, T. & Zagoskin, A.M. (2002a). Multi-terminal superconducting phase qubit, *Physica C*, 368, 310.

Amin, M.H.S., Omelyanchouk, A.N. & Zagoskin, A.M. (2001). Mesoscopic multiterminal Josephson structures. I. Effects of nonlocal weak coupling. *Low Temperature Physics*, 27, 616.

Amin, M.H.S., Omelyanchouk, A.N. & Zagoskin, A.M. (2002b). DC SQUID based on the mesoscopic multiterminal Josephson junction. *Physica C*, 178, 372-376.

Andrey, L. Pankratov (2002). Long Josephson junctions with spatially inhomogeneous driving, *Physical Review B*, 66, 134526.

Askerzade, I., Bozbey, A. & Cantürk, M. (2017). *Modern aspects of Josephson dynamics and superconductivity electronics* (Mathematical engineering), Springer, Berlin, 211 p.

Askerzade, I.N., Salati M.(2022). The influence of anharmonic current-phase relation on penetration depth in long Josephson junction. *Advanced Physical Research*, *4*(1), 16.

Askerzade, I.N. (2015) Effects of anharmonicity of current-phase relation in Josephson junctions. *Low Temperature Physics*, *41*(4), 241.

Askerzade, I.N., Kornev, V.K. (1994). Switching dynamics of the Goto pair of tunnel Josephson junctions. *Radiotekhnika i Elektronika*, *39*(5), 869.

Babajanov, D., Matyoqubov, H. & Matrasulov, D. (2018). Charged solitons in branched conducting polymers. *The Journal of Chemical Physics*, 149, 164908.

Barone, A., Paternò, G. (1982). *Physics and Applications of the Josephson Effect*, Wiley.

Barone, A., Esposito, F., Magee, C.J. & Scott, A.C. (1971). Theory and applications of the sine-gordon equation. *La Rivista del Nuovo Cimento*, *1*(2), 227–267.

Bennett, D., Bishop, A.R. & Trullinger, S.E. (1982). Coherence and chaos in the driven, damped sine-Gordon chain. *Zeitschrift für Physik B Condensed Matter*, 47, 265-277.

Burioni, R., Cassi, D., Sodano, P., Trombettoni, A. & Vezzani, A. (2006). Soliton propagation on chains with simple nonlocal defects. *Physica D: Nonlinear Phenomena*, 216, 71.

Burioni, R., Cassi, D., Sodano, P., Trombettoni, A. & Vezzani, A. (2005). Propagation of discrete solitons in inhomogeneous networks. *Chaos*, 15, 043501.

Caputo, J.-G., Dutykh, D. (2014). Nonlinear waves in networks: Model reduction for the sine-Gordon equation. *Physical Review E*, 90, 022912.

Caudrelier, V. (2015). On the inverse scattering method for integrable PDEs on a star graph. *Communications in Mathematical Physics*, 338, 893.

Cuevas-Maraver, J., Kevrekidis, P. & Williams, F. (2014). *The sine-gordon model and its applications: from pendula and Josephson junctions to gravity and high-energy physics*. Springer International Publishing.

Giuliano, D. & Sodano, P. (2009). Y-junction of superconducting Josephson chains. *Nuclear Physics B*, 811, 395.

Giuliano, D., Sodano, P. (2013). Realization of a two-channel Kondo model with Josephson junction networks. *EPL*, 103, 57006.

Golikova, T.E., Wolf, M.J., Beckmann, D., Batov, I.E., Bobkova, I.V., Bobkov, A.M. & Ryazanov, V.V. (2014). Nonlocal supercurrent in mesoscopic multiterminal SNS Josephson junction in the low-temperature limit. *Physical Review B*, 89, 104507.

Gul, Z., Ali, A. & Ahmad, I. (2018). Dynamics of ac-driven sine-Gordon equation for long Josephson junctions with fast varying perturbation. *Chaos, Solitons and Fractals,* 107, 103-110.

Heck, B. van, Mi, S. & Akhmerov, A.R. (2014). Single fermion manipulation via superconducting phase differences in multiterminal Josephson junctions, *Physical Review B*, 90, 155450.

Kairzhan, A., Pelinovsky, D.E. (2018). Spectral stability of shifted states on star graphs. *Journal of Physics A: Mathematical and Theoretical*, 51, 095203.

Likharev, K.K. (1986). *Introduction into Dynamics of Josephson Junctions and Circuits*. New York, Gordon Breach.

Malomed, B.A. (1989). Dynamics and kinetics of solitons in the driven damped double Sine-Gordon equation. *Physics Letters A*, 136, 395-401.

Matrasulov, D., Sabirov, K., Babajanov, D. & Susanto, H. (2020). Branched Josephson junctions: current carrying solitons in external magnetic fields. *EPL*, 130, 67002.

Matrasulov, D.U., Yusupov, J.R. & Sabirov, K.K. (2019). PT-symmetric quantum graphs. *Journal of Physics A: Mathematical and Theoretical*, 52, 155302.

Melin, R. (2022). Multiterminal ballistic Josephson junctions coupled to normal leads. *Physical Review B*, 105, 155418.

Noja, D. (2014). Nonlinear Schrödinger equation on graphs: recent results and open problems. *Philosophical Transactions of the Royal Society A*, 372, 20130002.

Noja, D., Pelinovsky, D. & Shaikhova, G. (2015). Bifurcations and stability of standing waves in the nonlinear Schrödinger equation on the tadpole graph. *Nonlinearity*, 28, 2343.

Nowak, M.P., Wimmer, M. & Akhmerov, A.R. (2019). Supercurrent carried by nonequilibrium quasiparticles in a multiterminal Josephson junction. *Physical Review B*, 99, 075416.

Pankratova, N., Lee, H., Kuzmin, R., Wickramasinghe, K., Mayer, W., Yuan, J., Vavilov, M.G., Shabani, J. & Manucharyan, V.E. (2020). Multiterminal Josephson effect. *Physical Review X*, 10, 031051.

Peralta Gavensky, L., Usaj, G. & Balseiro, C.A. (2023). Multi-terminal Josephson junctions: A road to topological flux networks. *EPL*, 141, 36001.

Riwar, R-P., Houzet, M., Meyer, J.S. & Nazarov, Y.V. (2016). Multi-terminal Josephson junctions as topological matter. *Nature*, 7, 11167.

Sabirov, K., Rakhmanov, S., Matrasulov, D. & Susanto, H. (2018). The stationary sine-Gordon equation on metric graphs: Exact analytical solutions for simple topologies. *Physics Letters A*, 382, 1092.

Sabirov, K.K., Akramov, M.E., Otajonov, R.Sh. & Matrasulov, D.U. (2020). Soliton generation in optical fiber networks. *Chaos, Solitons & Fractals*, 133, 109636.

Sabirov, K.K., Sobirov, Z.A., Babajanov, D. & Matrasulov, D.U. (2013). Stationary nonlinear Schrödinger equation on simplest graphs. *Physics Letters A*, 377, 860.

Sobirov, Z., Babajanov, D., Matrasulov, D., Nakamura, K. & Uecker, H. (2016). Sine-Gordon solitons in networks: Scattering and transmission at vertices. *EPL*, 115, 50002.

Sobirov, Z., Babajanov, D., Matrasulov, D., Nakamura, K. & Uecker, H. (2016). Sine-Gordon solitons in networks: Scattering and transmission at vertices. *EPL*, 115, 50002.

Sobirov, Z., Matrasulov, D., Sabirov, K., Sawada, S. & Nakamura, K. (2010). Integrable nonlinear Schrödinger equation on simple networks: Connection formula at vertices. *Physical Review E*, 81, 066602.

Susanto, H., van Gils, S., Doelman, A. and Derks, G. (2004). Semifluxons with a hump in a $0-\pi$ Josephson junction. *Physica C*, 408, 579.

Susanto, H., van Gils, S., Doelman, A., & Derks, G. (2004). Analysis on the stability of Josephson vortices at tricrystal boundaries: A $3\phi_0/2$ −flux case. *Physical Review B*, 69, 212503.

Yusupov, J.R., Matyokubov, Kh.Sh., Sabirov, K.K. & Matrasulov, D.U. (2020). Exciton dynamics in branched conducting polymers: Quantum graphs based approach. *Chemical Physics*, 537, 110861.

Zazunov, A., Egger, R., Alvarado, M. & Levy Yeyati, A. (2017). Josephson effect in multiterminal topological junctions. *Physical Review B*, 96, 024516.